



# GL-Model, Representing Emergence Of Dangerous State In Multiprocessor Management System

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**Abstract**— Graph-logic models (GL-models) are models of fault-tolerant multiprocessor systems' (FTMS) behavior in failure flow. Basis of the model is a graph which arcs are marked by some Boolean functions and the arguments of these functions are system processors states. GL-models are tools that allow efficiency increasing of reliability calculation of FTMS. The reliability calculation is based on the method of statistic experiments with GL-models. We propose the idea and method of creating GL-model, which answers the question if the multiprocessor complex object management system is technically (functionally) safe in the case when the failure of certain components in the system occurs. In other words, if there is a dangerous condition in the system, that is a state of the management system, in which at least one management function, that causes a dangerous condition, is not performed. The model can be used to conduct statistical experiments to determine the probability of transition to a dangerous state in case of system's degradation.

**Keywords**-Component; Functional Safety, Reliability, Fault-Tolerance, Multiprocessor Systems, Control.

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## I. INTRODUCTION

In recent years, the problem of calculating and provisioning the level of technical (functional) safety of complex systems, especially critical applications such as aircraft, nuclear power plants, space objects, medical and banking systems, is actively studied [1,2]. Such systems often include an object and its management system, and the safety of the complex depends on the state of both of these parts. Control systems often include multi-processor systems, a study of safety issues of which cannot be considered exhaustive at the moment. In [3,7] method of calculating the probability of the transition of the multiprocessor management system (MMS) into a dangerous state over time as its processor's failures happen is described. Such calculation can be done by performing statistical tests on relevant GL-models about which we will speak further. GL-models represent the reaction of a multiprocessor system (failure or operational state) to malfunctions of processors'. the design of GL-models which is of particular interest from a scientific and practical point of view.

## II. MANAGEMENT SYSTEM SAFETY

Security is one of the most important characteristics of any complex. An object of control can transit to dangerous state due to various reasons, among them - the lack of specific management functions, which are formed by a control system.

Such functions will be called dangerous. The control system, including MS, can be in several states by the theory of dependability [4], varying from fully operational to dangerous state, moving from one to another over time as its components fail. The state of MMS when one of the dangerous functions can not be formed will be called dangerous.

Such MS are, usually, configurable, i.e., after the failure of any processor, the processors remaining operational assume its functions. This fact shows that before the control system will go into a dangerous condition, it can survive a large number of failures, remaining in the operational or partially operational state.

## III. MODEL

It would be very convenient to use known GL-models of behaviors of fault-tolerant multiprocessor systems (FMS) in the flow of failures to construct the model referred in the introduction. Basic GL-model  $K(m,n)$  (model of the system, which transitions to a fault state when a failure occurs, the multiplicity of which exceeds the value of  $m$ ) is a graph [5] with a Boolean functions on its edges, which disappear in case when the corresponding functions equals to zero. Graph's connectivity reflects the state of the system (operational – faulty). Functions' variables – states of processors. Graph of

model K(m,n) loses its connectivity when the number of failures equals or exceeds m+1.

The dangerous state of the system caused by s failures can be simulated by constructing a GL-model of system K(s,n). In this case, the loss of connectivity in a model graph will correspond to the appearance of a dangerous condition in the system. However, the model turns out quite complicated, because complexity of edge functions and their number increases with increasing m, and s >> m, as previously stated, referring to the real value of the degree of fault tolerance.

Below is a different way, the idea of which is to reduce the problem of constructing a model of danger when s failures happen to the problem of constructing a GL-model of the behavior of FMS in the flow of failures in the case of K(n-s, n). It should be noted that if there are s failures in MMS, then n-s processors remain operational.

Just as in the m-fault-tolerant systems, the failure of FMS happens in the event of processors failure, exceeding m, we assume that the dangerous state in MMS occurs when as a result of MMS degradation (component failures) there are not more than m functional processors.

We construct a new model of K\*(m, n) by inverting all the variables in all the edges' functions of the model K(m, n), keeping the signs of Boolean operations and graph model unchanged.

#### IV. STATEMENT

Connectivity of the graph of model K\*(m, n) corresponds to the dangerous state of MMS.

Proof. In fact, the value of m in the model K\*(m, n) now corresponds to the number of operable processors, that is the appearance of n-m system failures. Indeed, each variable in the new K\*(m, n) model behaves opposite to how it behaves in the model K(m, n): value 1 (functional) in the model K\*(m, n) corresponds to the value 0 (failure) in the model K(m, n), and vice versa.

For simplicity, we write  $y_i = \overline{x_i}$ , each rib function  $f_i(X)$  of model K(m, n), in the model K\*(m, n) becomes a function  $f_i(Y)$ . In the model K\*(m, n) connectivity is lost (as well as in conventional GL-model) when the number of variables  $t^*$  equalling 0 exceeds the value m, that is,  $t^* > m$ . If  $t^* \leq m$ , connectivity takes place, and this corresponds to the emergence of a dangerous state in the system (since  $t^*$  is the number of operable processors). The proof is complete.

For illustration, consider the following example. Construct a GL-model of the system consisting of 8 processors and in which a dangerous condition occurs when there is not less than 5 failures, ie the model K\*(3,8). Following [5], the first step is forming ribs' functions of model K (3,8).

$$\begin{aligned} f1 &= x1+x2+x3x4 \\ f2 &= x1x2+x3+x4 \\ f3 &= (x1+x2)(x3+x4)(x1x2+x3x4)+x5x6x7x8 \\ f4 &= (x5+x6)(x7+x8)(x5x6+x7x8)+x1x2x3x4 \end{aligned}$$

$$f5 = x5+x6+x7x8$$

$$f6 = x5x6+x7+x8$$

Now we easily derive functions of model K\*(3,8).

$$f1^* = \overline{x_1} + \overline{x_2} + \overline{x_3} \overline{x_4}$$

$$f2^* = \overline{x_1} \overline{x_2} + \overline{x_3} + \overline{x_4}$$

$$f3^* = (\overline{x_1} + \overline{x_2})(\overline{x_3} + \overline{x_4})(\overline{x_1} \overline{x_2} + \overline{x_3} \overline{x_4}) + \overline{x_5} \overline{x_6} \overline{x_7} \overline{x_8}$$

$$f4^* = (\overline{x_5} + \overline{x_6})(\overline{x_7} + \overline{x_8})(\overline{x_5} \overline{x_6} + \overline{x_7} \overline{x_8}) + \overline{x_1} \overline{x_2} \overline{x_3} \overline{x_4}$$

$$f5^* = \overline{x_5} + \overline{x_6} + \overline{x_7} \overline{x_8}$$

$$f6^* = \overline{x_5} \overline{x_6} + \overline{x_7} + \overline{x_8}$$

We test the model on some vectors of system state with a different number of zero components. (look at the table below).

TABLE 1. VALUES OF RIBS' FUNCTIONS OF NEW MODEL K\*(3,8).

	x	x	x	x	x	x	x	x		f1*	f2*	f3*	f4*	f5*	f6*
	1	2	3	4	5	6	7	8							
v1	0	1	1	1	1	1	1	1		1	0	0	0	0	0
v2	0	0	1	1	1	1	1	1		1	1	0	0	0	0
v3	0	0	0	1	1	1	1	1		1	1	1	0	0	0
v4	0	0	0	0	1	1	1	1		1	1	1	1	0	0
v5	0	0	0	0	0	1	1	1		1	1	1	1	1	0
v6	0	0	0	0	0	0	1	1		1	1	1	1	1	1

The last two vectors indicate the appearance of dangerous state, as the connectivity appears in the graph (either all the edges are present, or only one is absent - the vector v5). This is correct, because in this system a dangerous condition occurs when not more than 3 processors remain operable.

Note that the above text was about basic models. If the dangerous state occurs in other combinations of operable processors, the modification should be carried out to model K\*(m, n) using one of the methods described in [6].

For example, if in this case dangerous state occurs when processors 5,6,7 and 8 Remain operable, then it is possible to draw inner rib with its function g, as shown on the figure, to reestablish connectivity in the graph, despite that ribs with

functions f\*5 and f\*6 are missing:  $g = \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} \cdot \overline{x_4} \cdot \overline{x_5}$

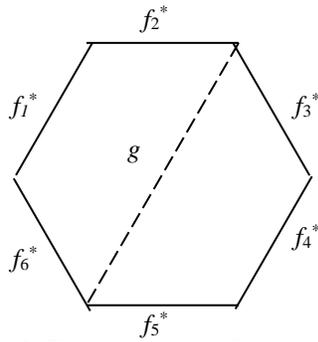


Figure 1. Example of modification of the base model

## V. CONCLUSION

In this paper, a GL-model of  $K^*(m, n)$  was proposed, showing the appearance of a multiprocessor complex objects management system dangerous state. It refers to a state when at least one of the control functions, the lack of which leads to a dangerous condition of the control object, can not be formed, in particular, in the case when in the system of  $n$  processors  $m$  or less processors remain operable. It is shown that modification of the model  $K^*(m, n)$  may be carried out based on known modification methods of GL-models  $K(m, n)$  of multiprocessor systems failover behavior in failure stream. This applies to cases when, for example, the system transitions into dangerous state with the number of operable processors exceeding  $m$ .

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